PC– 361 CV-19

M.A./M.Sc. Mathematics (III SEM.)
Examination Dec.-2020
Compulsory
Paper-II
"PARTIAL DIFFERENTIAL EQUATIONS, MECHANICS AND GRAVITATION-I"

Time: Three Hours] [Maximum Marks: 80
[Minimum Pass Marks: 29

Note: Answer from Both Section as Directed. The figures in the right hand margin indicate marks.

Section –A

1. Answer the following questions: 1X10
   a. Find the Laplace transform of the function F(t)=1
   b. Write down the Laplace first shifting Theorem.
   c. What is the value of $L^{-1}\{a/p^2+a^2\}$?
   d. Write down the heat conduction Equation.
   e. Write down the two dimensional wave equation
   f. Write down the Greens function for Laplace Equation.'
   g. What is the attraction of their uniform spherical Shell at an internal point?
   h. Write down Poisson equation.
   i. Write down the mean value theorem for harmonic functions.
   j. Write down the dimensional wave equation.
2. Answer the following questions: 2x5
   a. Find the Laplace transform of the function $F(t) = sint\,cost$
   b. Find the Laplace transform of the function $F(t) = sin\sqrt{t}$
   c. If Φ is harmonic function in R1 and $\partial\Phi/\partial\eta$=0 on R_2 then Φ is Constant in $\bar{R}$
   d. Write down the statement of Gauss theorem.
   e. What is the attraction of rod, which is of infinite length.

Section –B

12x5

3. Answer all question:
   a. If $F(t)$ is a function of class A and if $L\,[F(t) = f(p)]$ then show that
   $L\,[t^n F(t)] = (-1)^n \frac{d^n}{dp^n} f(p)$ Where $n = 1,2,3, \ldots\ldots\ldots n$
   b. Find Laplace transform of the function $F(t) = 4/p^2 - 6p + 25$
   OR
   Solve the equation $\partial u/\partial t = 2\,\partial^2 u/\partial x^2$
   If U(0,t)=0, u(x,0)= e^{-x} x>0, t>0, u(x,t) is bounded where x>0, t>0 (By using Laplace Transform)
4. a. Solve the differential equation :
   $x^2p+y^2q=z^2$
   b. Solve the differential equation $xzp + yzq = xy$

   OR

   a. Solve the differential Equation
   $x(y^2+z)p-y(x^2+z)q=z(x^2-y^2)$
   b. Solve the differential equations:

$$px + qy = z\sqrt{(1 + pq)}$$

5. Describe the two dimensional wave equation.

OR

Find the solution of two dimensional heat equation (by using the method of separation of variables)

6. To find the attraction of spherical shell of finite thickness, bounded by the spheres of radii a and b.

OR

Find the attraction of a solid sphere of mass M and radian a.

7. Show that the attraction of a circular disc of radious a, where law of attraction is $\mu/(\text{distance})^3$, in $M\mu/C(c^2-a^2)$ or $M\mu a/a^2(a^2-c^2)$
According as $c \geq a$.
Where M is the mass of the disc and c the distance from the centre of the attracted point which is in the plane of the disc.

OR

Prove that a solid uniform hemisphere of radius a , exerts no resultant attraction at a point on its axis at a distance C from the centre given by the equation.

$$12c^4-8a^3c+3a^4=0$$